Int. J. Heat Mass Transfer. Vol. 33, No. 3, pp. 457–464, 1990
Printed in Great Britain

0017-9310/90 $3.00+0.00

Cooling of a continuous moving sheet of finite thickness in the presence of natural convection

S. L. LEE and J. S. TSAI
Department of Power Mechanical Engineering, National Tsing-Hua University, Hsinchu, Taiwan 30043, Republic of China

(*Received* 24 *January* 1989)

Abstract—The present investigation studies the cooling of a continuous moving sheet of finite thickness. The effect of the buoyancy force is also taken into account. The temperature distribution along the solid–fluid interface is determined by solving a conjugate heat transfer problem that consists of heat conduction inside the sheet and induced mixed convection adjacent to the sheet surface. For a better numerical stability, the weighting function scheme along with an axial coordinate transformation is employed to solve the transformed boundary layer equations. Three parameters are found to exist in the present investigation. They are the Prandtl number of the fluid Pr, the buoyancy parameter Ω and the heat capacity ratio C. Numerical results including the Biot number, the surface temperature and the overall heat transfer rate of the sheet are presented for $0.7 \leqslant Pr \leqslant 100$, $0 \leqslant \Omega \leqslant 10$ and $0.1 \leqslant C \leqslant 1$. The buoyancy force is seen to have a significant effect on the results. The heat capacity ratio, however, is the most important parameter. Based on the present results, it is concluded that using a liquid as the cooling medium could obtain a better cooling performance than using a gas. This is because the liquid has a larger heat capacity than a gas. The Prandtl number has only a minor effect.

INTRODUCTION

IT IS A common method to draw a hot material through a slot (or an orifice) in sheet (or fibre) manufacturing. In these industrial processes, control of the cooling rate of the sheets (or the fibres) is very important to obtain a desired material structure. As the continuous sheets (or fibres) move through a cooling tank or atmosphere, they are cooled by the boundary layers induced on their surfaces due to the viscous force. The induced boundary layers were found to dominate the cooling rate of the sheets (or fibres) and have been extensively studied by many investigators for continuous moving sheets [1–14] and cylinders [1, 3, 15–21].

Sakiadis [1, 2] was the first investigator to analyse the boundary layer flow induced on a continuous moving sheet. Later, his predicted velocity profile was verified by the experiments performed by Tsou *et al.* [6, 7]. This indicates that the mathematically described boundary layer on a continuous moving surface is a physically realizable flow. Tsou *et al.* [6] found also that the critical Reynolds number is about 4.96×10^6 on a continuous moving sheet as compared to 0.949×10^5 on a classic Blasius flow. Hence, the induced boundary layer flow on a moving sheet is practically laminar. In their experimental study on cooling time of silica fibres, Arridge and Prior [16] found that the temperatures of the silica fibres followed Newton's cooling law as they decreased from 1750 to 150°C at a moving speed of 1.78–5.08 m s^{-1}. It was thus believed that the radiative heat transfer is essentially negligible as compared to the forced convection even though the temperature of the moving surface could be as high as 1750°C. This finding was later verified numerically by Bourne and Dixon [21].

It is noted that the surface temperature of the moving surface was assumed uniform in the previous studies [4, 7, 8, 11–14]. As pointed out by Tsou *et al.* [7] and Moutsoglou and Chen [12], changing the boundary condition of a moving surface from uniform wall temperature to uniform wall heat flux could increase the heat transfer coefficient by 30–70%. Unfortunately, the surface condition of the continuous moving sheet is neither uniform wall temperature nor uniform surface heat flux. To evaluate the surface temperature of a moving cylinder, Griffith [17] solved the heat conduction problem inside the cylinder by means of Duhamel's theorem. The heat transfer coefficient $h_0(x)$ used in the calculation was estimated from results based on the uniform wall temperature case. Such a treatment cannot observe the effect of the heat capacity ratio of the materials and the ambient fluid. In addition, Griffith's analysis is restricted to the conditions of $Pr = \infty$ and $k = k_s$. Another attempt to study the surface temperature of a moving cylinder was made by Kuiken [18]. However, his analysis is limited to $4x/(Pe_s D) \ll 1$ because of the use of penetration theory in solving the heat conduction equation inside the cylinder. To investigate the effect of heat capacity ratio of the sheet and the surrounding fluid, Erickson *et al.* [5] assumed that the thermal conductivity of the sheet material is infinite and/or the thickness of the sheet is sufficiently small such that the temperature of the sheet depends only on the axial coordinate, i.e. $T_s = T_s(x)$. This same assumption was also employed by Kuiken [9] and Bourne and Dixon [21].

NOMENCLATURE

b	decaying coefficient defined in equation (13)
$Bi(\xi)$	Biot number of the moving sheet, h_0H/k_s
C	heat capacity ratio of the fluid and the solid, $[(k\rho c_p)/(k\rho c_p)_s]^{1/2}$
c_p	specific heat at constant pressure [J kg^{-1} K^{-1}]
D	diameter of cylinder [m]
f	transformed stream function, $\psi\delta/\nu x$
g	gravitational acceleration, 9.81 m s^{-2}
Gr_H	Grashof number, $g\beta\|T_0-T_\infty\|H^3/\nu^2$
H	thickness of the moving sheet [m]
h_0	cooling coefficient on the surface of the moving sheet, $Q_w/(T_w-T_\infty)$
k	thermal conductivity of the fluid [W m^{-1} K^{-1}]
k_s	thermal conductivity of the sheet [W m^{-1} K^{-1}]
Pe_s	Peclet number of the moving sheet, u_0H/α_s
Pr	Prandtl number of the fluid, ν/α
$q(\xi)$	overall heat transfer rate of the sheet defined by equation (9)
Q_w	heat flux on the surface of the moving sheet [W m^{-2}]
Re_H	Reynolds number, u_0H/ν
Re_x	Reynolds number, u_0x/ν
T	temperature of the fluid [K]
T_s	temperature inside the sheet [K]
T_0	temperature of the moving sheet at $x = 0$ [K]
T_∞	temperature of the fluid at $y = \infty$ [K]
TOL	prescribed tolerance of numerical error defined in equation (12)
u_0	drawing speed of the continuous sheet [m s^{-1}]
x	axial coordinate with the slot as the origin [m]
y	transverse coordinate measured from the centerline of the sheet [m]
z	transformed axial coordinate, $1-\exp(-b\xi)$.

Greek symbols

α	thermal diffusivity of the fluid [m^2 s^{-1}]
α_s	thermal diffusivity of the sheet [m^2 s^{-1}]
β	volumetric coefficient of thermal expansion [K^{-1}]
δ	characteristic boundary layer thickness, $x(Re_x)^{-1/2}$
η	transformed transverse coordinate, $2y/H-1$ if $y < H/2$ and $(y-H/2)/\delta$ if $y \geq H/2$
θ	dimensionless temperature of the fluid, $(T-T_\infty)/(T_0-T_\infty)$
$\theta_w(\xi)$	$\theta(\xi, 0)$
ν	kinematic viscosity [m^2 s^{-1}]
ξ	transformed axial coordinate, $4x/(Pe_sH)$
ρ	density of the fluid [kg m^{-3}]
ρ_s	density of the sheet material [kg m^{-3}]
σ	index, 1 for buoyancy assisting flow and -1 for buoyancy opposing flow
ϕ	dimensionless temperature inside the moving sheet, $(T_s-T_\infty)/(T_0-T_\infty)$
$\phi_w(\xi)$	$\phi(\xi, 0)$
ψ	stream function [m^2 s^{-1}]
Ω	buoyancy parameter, $(Gr_H/Re_H^2)(Pe_s/4)$.

Aside from refs. [8, 11, 12], the effect of the buoyancy force resulting from the temperature differences in the fluid were neglected in all of the previous studies. In their experimental work, Griffin and Throne [8] employed an isothermal belt that moved through a surrounding air of 75 F while the surface temperature of the belt was essentially held at 175 F. The Reynolds number was less than 60 000 such that the boundary layer flow in this experiment was laminar. Due to the buoyancy effect, the measured Nusselt number values were found to be 10–60% larger than the prediction without the buoyancy effect [4]. Recently, Chen and co-workers [11, 12] considered the buoyancy effects in a boundary layer induced by a continuous moving isothermal sheet by the use of the local non-similarity method. Their Nusselt number results seemed to agree with measurements [8] to within the experimental error.

In the present study, cooling of a continuous moving sheet of finite thickness is investigated. Conjugate heat transfer consisting of the non-similar thermal boundary layer and the heat conduction inside the moving sheet will be solved by using the weighting function scheme proposed in ref. [22]. In the solution procedure, the temperature distribution along the solid–fluid interface is guessed such that the temperatures in fluid and solid regions can be solved separately. The interface temperature then is adjusted until the energy conservation law on the solid–fluid interface is satisfied within a prescribed tolerance. Such a conjugate heat transfer problem (a non-similar thermal boundary layer and a solid of finite thickness) has not been studied in the past.

THEORETICAL ANALYSIS

Consider a continuous flat sheet that has a finite thickness H. The sheet originates from a slot and is moving vertically with a constant velocity u_0 through an otherwise quiescent fluid at temperature T_∞. Due to the no-slip condition of a viscous fluid on a solid boundary, boundary layer flows will be induced on both sides of the flat sheet. As in conventional studies, the positive x coordinate is measured along the direction of the moving sheet with the slot as the origin. The positive y having a direction normal to the sheet surface is measured from the centreline of the sheet. The moving speed of the flat sheet is assumed sufficiently large such that the axial conduction inside the sheet is negligible. This assumption leads also to a uniform temperature distribution across the thickness of the sheet at $x = 0$, i.e. $T_s(0, y) = T_0$. After introducing the dimensionless transformation

$$\xi = 4x/(Pe_s H), \quad \eta = 2y/H-1$$
$$\phi = (T_s-T_\infty)/(T_0-T_\infty) \tag{1}$$

the heat conduction problem inside the flat sheet becomes

$$\partial\phi/\partial\xi = \phi''$$
$$\phi(0,\eta) = 1, \quad \phi'(\xi,-1) = 0, \quad \phi(\xi,0) = \phi_w(\xi) \tag{2}$$

where the primes denote partial differentiation with respect to η. The boundary condition at $\eta = -1$ is assigned insulated owing to the symmetry of the thermal boundary layers adjacent to both sides of the vertical flat sheet. The temperature distribution along the sheet surface $\phi_w(\xi)$ will be determined such that an energy balance equation at the interface of the flat sheet and the ambient fluid is satisfied.

As demonstrated in Appendix A of ref. [23], by introducing the dimensionless transformation

$$\psi = \nu x f(\xi,\eta)/\delta, \quad \xi = 4x/(Pe_s H), \quad \eta = (y-H/2)/\delta$$
$$\delta(x) = x(Re_x)^{-1/2}, \quad \theta = (T-T_\infty)/(T_0-T_\infty) \tag{3}$$

the conservation equations for the induced boundary layer flows become

$$f'''+\tfrac{1}{2}ff''+\sigma\Omega\xi\theta = \xi(f'\partial f'/\partial\xi-f''\partial f/\partial\xi)$$
$$\theta''+\tfrac{1}{2}Pr\, f\theta' = Pr\,\xi(f'\partial\theta/\partial\xi-\theta'\partial f/\partial\xi). \tag{4}$$

Again, the primes stand for partial differentiation with respect to η. The associated boundary conditions are now

$$f(\xi,0) = f'(\xi,0)-1 = f'(\xi,\infty) = 0$$
$$\theta(\xi,0) = \theta_w(\xi), \quad \theta(\xi,\infty) = 0. \tag{5}$$

In equation (4), $\sigma = 1$ stands for buoyancy assisting flow and $\sigma = -1$ for buoyancy opposing flow. The buoyancy parameter Ω is defined by

$$\Omega = (Gr_H/Re_H^2)(Pe_s/4). \tag{6}$$

It should be noted here that $\xi = 4x/(Pe_s H)$ has been used in both dimensionless transformations (1) and (3) such that a compatible transformed axial coordinate between the flat sheet and the boundary layer flow can be achieved. The energy conservation law at the interface of the flat sheet and the fluid can be expressed as

$$\phi_w(\xi) = \theta_w(\xi)$$
$$\phi'(\xi,0)/\theta'(\xi,0) = C(Pr\,\xi)^{-1/2} \tag{7}$$

where $C = [(k\rho c_p)/(k\rho c_p)_s]^{1/2}$ is the heat capacity ratio of the fluid and the continuous moving sheet.

In summary, it is concluded that equations (2), (4), (5) and (7) constitute a conjugate heat transfer problem. For given values of σ, Ω, Pr and C, there uniquely exists a solution for $\phi(\xi,\eta)$ and $\theta(\xi,\eta)$. Once the conjugate heat transfer problem is solved, the physical quantities of interest such as the Biot number distribution $Bi(\xi)$ and the dimensionless overall heat transfer rate $q(\xi)$ are evaluated by

$$Bi(\xi) = h_0 H/k_s = -(2/\phi_w)\phi'(\xi,0) \tag{8}$$

$$q(\xi) = 1-\int_{-1}^{0}\phi(\xi,\eta)\,d\eta \tag{9}$$

where the heat transfer coefficient h_0 is defined by

$$Q_w(x) = h_0(T_w-T_\infty) = -k_s\partial T_s(x,H/2)/\partial y. \tag{10}$$

Using equations (7), the Biot number (8) can also be determined by

$$Bi(\xi) = -2C(Pr\,\xi)^{-1/2}\theta'(\xi,0)/\theta_w(\xi). \tag{11}$$

It is important to note that when $(\rho c_p)/(\rho c_p)_s \ll 1$, the value of the heat capacity ratio C becomes zero. This leads to an insulation condition for the moving sheet ($Bi(\xi) = 0$) as observable from equation (11). Thus, the interface temperature can be assumed uniform $\phi_w(\xi) = \theta_w(\xi) = 1$ under this particular situation. Such an assumption greatly simplifies the problem and thus has been widely employed by previous investigators [4, 7, 8, 11–14]. However, $C = 0$ and $Bi(\xi) = 0$ could arise also from the condition $k/k_s \ll 1$ when the moving sheet has a very large thermal conductivity. Unfortunately, this condition ($C = 0$ and $Bi = 0$) does not imply $T_s = T_s(x)$ when $(\rho c_p)/(\rho c_p)_s$ has a finite value. Therefore, the analyses performed previously [5, 9, 21] do not satisfy equation (8), because they assumed a non-zero Biot number distribution when $k_s = \infty$.

METHOD OF SOLUTION

Before solving equations (2), (4) and (5), the dimensionless temperature distribution along the solid–liquid interface $\phi_w(\xi)$ or $\theta_w(\xi)$ must be guessed. Fortunately, both heat transfer problems in equations (2) and in equations (4) and (5) are of parabolic type. One thus needs to guess only a single value for $\phi_w(\xi)$ at the 'present' location ξ during the solution procedure from $\xi = 0$ to ∞. Based on the guessed $\phi_w(\xi)$ value, equations (2), (4) and (5) are solved and the temperature gradients $\phi'(\xi,0)$ and $\theta'(\xi,0)$ are evaluated from the updated solution of $\phi(\xi,\eta)$ and $\theta(\xi,\eta)$. The value of $\phi_w(\xi)$ then is refined and the numerical procedure is repeated until the energy balance equation (7) is satisfied within a prescribed tolerance TOL, i.e.

$$|\phi'(\xi,0)/\theta'(\xi,0)-C(Pr\,\xi)^{-1/2}| \leqslant TOL. \quad (12)$$

This numerical procedure seems to be quite straightforward. However, as can be seen from equation (12), the value of $\phi'(0,0)$ should have an infinite value at the singular point $\xi = 0$. Numerical instability thus might arise from this particular point if a coarse step size $\Delta\xi$ is employed. Theoretically speaking, the local Biot number at $\xi = 0$ should be infinite, because the thickness of the boundary layer is zero there. The value of the local Biot number, however, decreases rapidly along the axial direction due to an increasing boundary layer thickness. Thus, small step size $\Delta\xi$ is required in the region of $\xi \approx 0$.

To remedy this numerical difficulty, the axial coordinate ξ was transformed onto the z-coordinate as suggested by Lee [24], i.e.

$$z = 1-\exp(-b\xi), \quad \xi = -\frac{1}{b}\ln(1-z)$$

$$\partial/\partial\xi = b(1-z)\partial/\partial z \quad (13)$$

where the decaying coefficient b is to be assigned such that a desired grid system in the ξ-coordinate can be generated, while the grid in the z domain is uniformly distributed.

Numerical difficulties might also arise in solving the system of coupled, non-linear partial differential equations (4) and (5). As mentioned earlier, the value of $\theta_w(z)$ or $\phi_w(z)$ will be guessed in the solution procedure. However, the error in the guessed $\theta_w(z)$ could result in a diverging result, especially when a large value is assigned to the heat capacity ratio C such that the function $\theta_w(z)$ has a strong variation on z. This numerical difficulty will become even more serious as the Prandtl number has a large value. For a large Prandtl number, the thickness of the thermal boundary layer is very small compared to that of the momentum boundary layer. Such a situation is known as a 'stiff boundary layer'. Fortunately, the weighting function scheme proposed in ref. [22] along with the axial coordinate transformation (13) was found to solve the stiff problems very efficiently in the present investigation.

RESULTS AND DISCUSSION

Numerical results were obtained for the case of buoyancy assisting flow ($\sigma = 1$) for Prandtl numbers of 0.7, 7 and 100. They cover the buoyancy parameters $\Omega = 0$, 0.1, 1, 5 and 10 and the heat capacity ratios $C = 0.1$, 0.2, 0.5 and 1.0 for each Prandtl number. The domain of computations was $0 \leqslant \eta \leqslant 10$ and $0 \leqslant \xi \leqslant 5.037$. The decaying coefficient b employed in equation (13) was 0.4. The step size $\Delta\eta = 0.05$ and $\Delta z = 1/150$ was found to be adequate for all parameters that were investigated in the present study. All computations were performed on a CDC Cyber 840 computer.

As an illustration, the velocity profiles for $Pr = 7$,

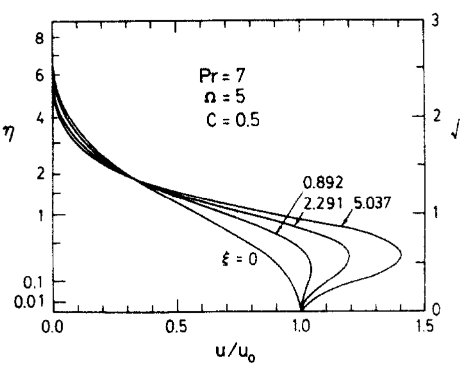

FIG. 1. The velocity profile in the boundary layer flow for the case of $Pr = 7$, $\Omega = 5$ and $C = 0.5$.

$\Omega = 5$ and $C = 0.5$ are plotted in Fig. 1 for various axial locations. The buoyancy effects were found to exist in a thin layer adjacent to the wall ($0 \leqslant \eta \leqslant 1$). Thus, in Fig. 1, the velocity profiles are presented vs the normal coordinate $\eta^{1/2}$ for a better observation on the buoyancy effects. Note that the non-similar boundary layer equation (4) reduces to the Blasius equation if the buoyancy effects are neglected ($\Omega = 0$). The velocity thus possesses similarity solution under this particular situation. Note also that the buoyancy effects vanish at the slot region as can be seen by substituting $\xi = 0$ into equation (4). Therefore, the curve labelled with $\xi = 0$ in Fig. 1 represents also the case of no buoyancy effect. From Fig. 1, one observes that the velocity profile overshoots by 40% at $\xi = 5.037$. This implies that the buoyancy force could have a significant effect on the heat transfer.

Figure 2 reveals the temperature variations for the same parameters employed in Fig. 1. The region $-1 \leqslant \eta \leqslant 0$ stands for the temperature $\phi(\xi,\eta)$ inside the continuous sheet, whereas the region $\eta \geqslant 0$ stands for the temperature of the fluid. The position $\eta = 0$ is the solid–fluid interface. It is important to note that the interface temperature $\phi_w(\xi)$ decreases rapidly in a region near the slot ($\xi \approx 0$) due to a small thermal boundary layer thickness. However, the temperature at the centreline of the sheet does not have a fast response to the cooling process. Therefore, it is

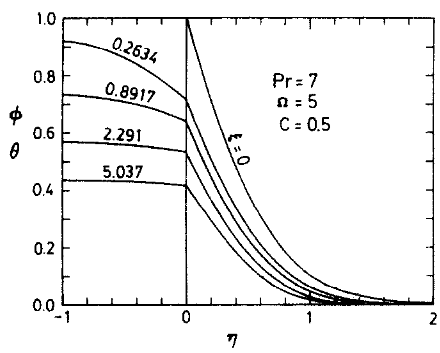

FIG. 2. The temperatures inside the sheet ($\eta \leqslant 0$) and in the boundary layer ($\eta \geqslant 0$) for the case of $Pr = 7$, $\Omega = 5$ and $C = 0.5$.

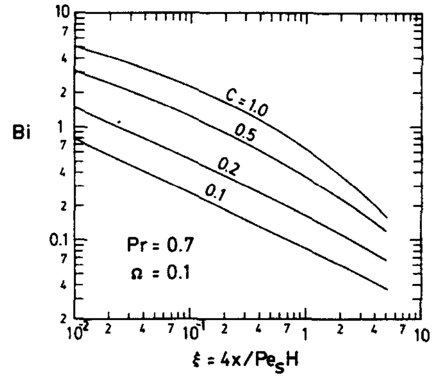

FIG. 3(a). Effect of heat capacity ratio on the Biot number for $Pr = 0.7$ and $\Omega = 0.1$.

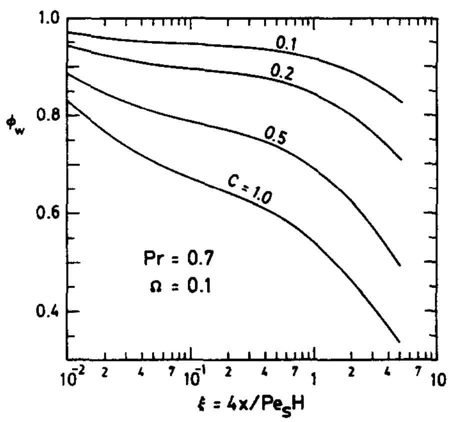

FIG. 3(b). Effect of heat capacity ratio on the surface temperature of the sheet for $Pr = 0.7$ and $\Omega = 0.1$.

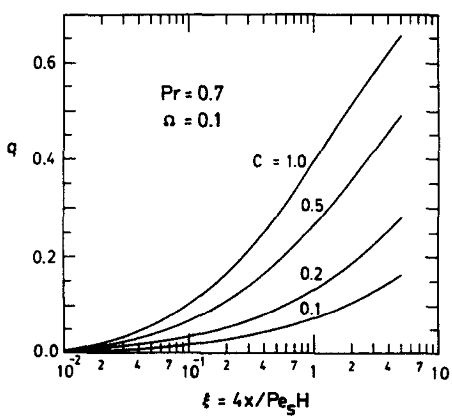

FIG. 3(c). Effect of heat capacity ratio on the overall heat transfer rate for $Pr = 0.7$ and $\Omega = 0.1$.

improper to assume $T_s = T_s(x)$ if the value of ξ is small in the entire observation length. This phenomenon exists when the moving speed and/or the thickness of the sheet have large values such that the value of the parameter $(Pe_s H)/4$ becomes very large.

The effects of the capacity ratio C on the Biot number $Bi(\xi)$, the surface temperature $\phi_w(\xi)$ and the overall heat transfer rate $q(\xi)$ are shown, respectively, on Figs. 3(a)–(c) for the case of $Pr = 0.7$ and $\Omega = 0.1$.

It is interesting to note from Fig. 3(a) that the variation of the Biot number along the axial coordinate ξ approximates a steep straight line in full logarithmic graph paper. This means that the Biot number possesses a strong power-law variation along the axial direction. For instance, for the case $C = 1.0$, the Biot number decreases from 5.03 at $\xi = 0.01$ to 0.161 at $\xi = 5$. The Biot number increases greatly when the heat capacity ratio increases. The large Biot number value existing in the slot region ($\xi \approx 0$) causes a rapid decrease in the surface temperature $\phi_w(\xi)$ in the region $\xi < 0.1$ as can be observed from Fig. 3(b). This effect becomes more pronounced when the value of the heat capacity ratio is increased. Nevertheless, the temperature at the centreline of the sheet could be still high (see Fig. 2). This results in a large transverse temperature gradient inside the sheet. Such a strong temperature variation is believed to have a significant effect on the material structure. As expected, it can be observed from Fig. 3(c) that the heat capacity ratio C has a considerable effect on the heat transfer rate (the fraction of heat removed). This finding is consistent with physical reasoning.

Figures 4(a)–(c), respectively, show the effect of

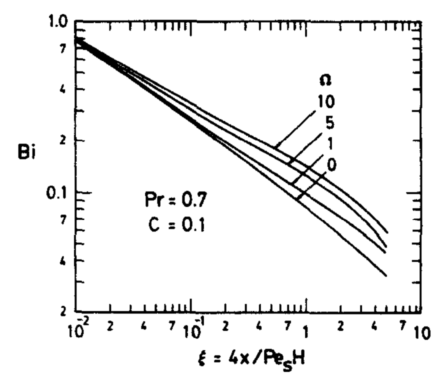

FIG. 4(a). Effect of buoyancy force on the Biot number for $Pr = 0.7$ and $C = 0.1$.

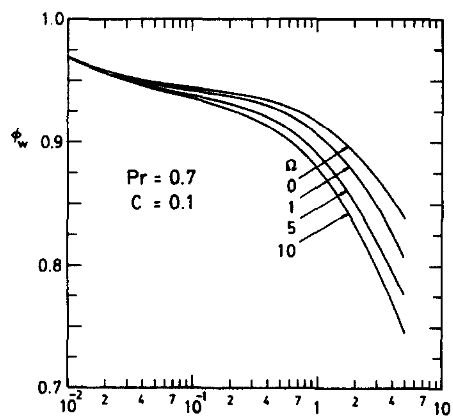

FIG. 4(b). Effect of buoyancy force on the surface temperature of the sheet for $Pr = 0.7$ and $C = 0.1$.

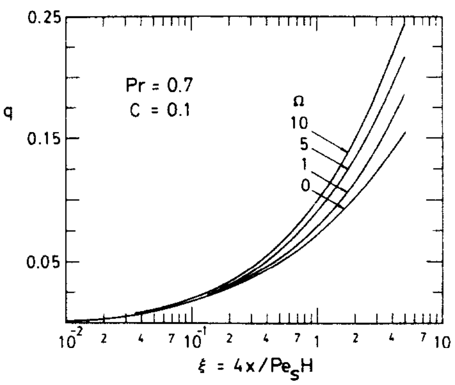

FIG. 4(c). Effect of buoyancy force on the overall heat transfer rate for $Pr = 0.7$ and $C = 0.1$.

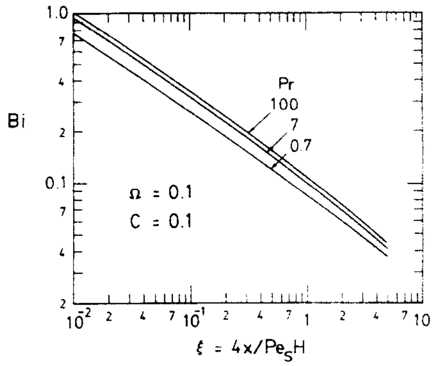

FIG. 5(a). Effect of Prandtl number on the Biot number for $\Omega = 0.1$ and $C = 0.1$.

the buoyancy parameter Ω on the Biot number, the surface temperature and the overall heat transfer rate for the case of $Pr = 0.7$ and $C = 0.1$. From Fig. 4(a), one sees that the curves of the Biot number based on various Ω-values merge into the one without the buoyancy effect ($\Omega = 0$) at $\xi \approx 0$. This is because the buoyancy effects vanish there as mentioned earlier. In downstream locations, however, an assisting buoyancy force could increase the velocity (see Fig. 1) by 40% or more and thus could enhance the heat transfer by a large amount. For example, for the case $\Omega = 10$ and the location $\xi = 5$, the buoyancy force increases the Biot number from 0.0330 to 0.0600, decreases the surface temperature from 0.840 to 0.748 and increases the overall heat transfer rate from 0.1546 to 0.2439. These effects can be observed, respectively, from Figs. 4(a) to (c).

Finally, the influences of the Prandtl number on the Biot number, the surface temperature and the overall heat transfer rate are presented, respectively, in Figs. 5(a)–(c). From these figures, an increase in the Prandtl number is found to increase the Biot number, decrease the surface temperature and thus increase the overall heat transfer rate. However, the effect of the Prandtl number is far less significant than that of the heat capacity ratio (see Figs. 3(a)–(c)). In general, liquid has a larger Prandtl number and a larger heat capacity ($k\rho c_p$) than a gas. Therefore, using a liquid as the cooling medium a better cooling performance could be obtained than using a gas. Based on the present results, it is concluded that most of the good cooling performance of a liquid comes from its large heat capacity.

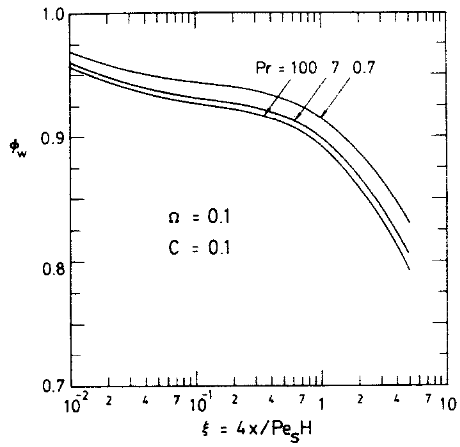

FIG. 5(b). Effect of Prandtl number on the surface temperature of the sheet for $\Omega = 0.1$ and $C = 0.1$.

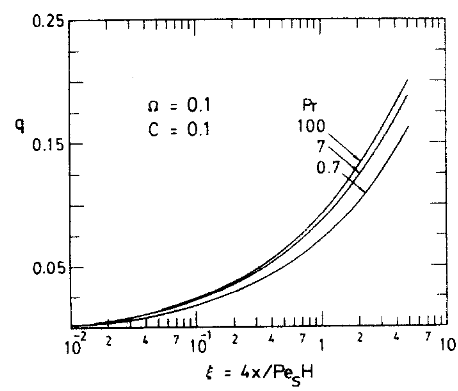

FIG. 5(c). Effect of Prandtl number on the overall heat transfer rate for $\Omega = 0.1$ and $C = 0.1$.

CONCLUSION

Conjugate heat transfer consisting of heat conduction inside a continuous moving sheet and mixed convection in the induced boundary layer flow is investigated by employing the weighting function scheme. When a coarse step size is used, numerical instability could arise in a region near the slot ($\xi \approx 0$) because of a rapid decrease in the temperature of the solid–fluid interface. To remedy this numerical difficulty, a coordinate transformation is performed in the axial direction. From the numerical results, the buoyancy force is seen to have a significant effect on the Biot number, surface temperature of the moving sheet and the overall heat transfer rate. The heat capacity ratio, however, is the most important parameter in the present conjugate heat transfer problem.

A large heat capacity ratio results in a large Biot number. This causes a rapid decrease in the surface temperature of the sheet in the region $\xi < 0.1$. Thus, it is improper to assume a uniform temperature distribution across the thickness of the moving sheet in that region. In general, a liquid has a larger Prandtl number and a larger heat capacity than a gas. Therefore, using a liquid as the cooling medium a better cooling performance could be obtained than using a gas. Based on the present results, it is concluded that most of the good cooling performance of a liquid comes from its large heat capacity.

Acknowledgement—The authors wish to express their appreciation to the National Science Council of the Republic of China in Taiwan for the financial support of this work through the project NSC78-0401-E007-12.

REFERENCES

1. B. C. Sakiadis, Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow, *A.I.Ch.E. Jl* **7**, 26–28 (1961).
2. B. C. Sakiadis, Boundary-layer behavior on continuous solid surfaces: II. Boundary-layer on a continuous flat surface, *A.I.Ch.E. Jl* **7**, 221–225 (1961).
3. E. A. Koldenhof, Laminar boundary layers on continuous flat and cylindrical surfaces, *A.I.Ch.E. Jl* **9**, 411–418 (1963).
4. L. E. Erickson, L. T. Fan and V. G. Fox, Heat and mass transfer on a moving continuous flat plate with suction or injection, *Ind. Engng Chem. Fundam.* **5**, 19–25 (1966).
5. L. E. Erickson, L. C. Cha and L. T. Fan, The cooling of a moving continuous flat sheet, *Chem. Engng Prog. Symp. Ser.* **62**, 157–165 (1966).
6. F. K. Tsou, E. M. Sparrow and E. F. Kurtz, Hydrodynamic stability of the boundary layer on a continuous moving surface, *J. Fluid Mech.* **26**, 145–161 (1966).
7. F. K. Tsou, E. M. Sparrow and R. J. Goldstein, Flow and heat transfer in the boundary layer on a continuous moving surface, *Int. J. Heat Mass Transfer* **10**, 219–235 (1967).
8. J. F. Griffin and J. L. Throne, On thermal boundary layer growth on continuous moving belts, *A.I.Ch.E. Jl* **13**, 1210–1211 (1967).
9. H. K. Kuiken, The cooling of a low-heat-resistance sheet moving through a fluid, *Proc. R. Soc. London* **A341**, 233–252 (1974).
10. H. K. Kuiken, Solidification of a liquid on a moving sheet, *Int. J. Heat Mass Transfer* **20**, 309–314 (1977).
11. T. S. Chen and F. A. Strobel, Buoyancy effects in boundary layer adjacent to a continuous, moving horizontal flat plate, *ASME J. Heat Transfer* **102**, 170–172 (1980).
12. A. Moutsoglou and T. S. Chen, Buoyancy effects in boundary layers on inclined, continuous, moving sheets, *ASME J. Heat Transfer* **102**, 371–373 (1980).
13. T. A. Abdelhafez, Skin friction and heat transfer on a continuous flat surface moving in a parallel free stream, *Int. J. Heat Mass Transfer* **28**, 1234–1237 (1985).
14. N. Ramachandran, B. F. Armaly and T. S. Chen, Correlation for laminar mixed convection in boundary layers adjacent to inclined, continuous moving sheets, *Int. J. Heat Mass Transfer* **30**, 2196–2199 (1987).
15. B. C. Sakiadis, Boundary-layer behavior on continuous solid surfaces: III. The boundary layer on a continuous cylindrical surface, *A.I.Ch.E. Jl* **7**, 467–472 (1961).
16. R. G. C. Arridge and K. Prior, Cooling time of silica fibers, *Nature* **203**, 386–387 (1964).
17. R. M. Griffith, Velocity, temperature and concentration distributions during fiber spinning, *Ind. Engng Chem. Fundam.* **3**, 245–250 (1964).
18. H. K. Kuiken, The cooling of a cylinder moving through a fluid, *Appl. Scient. Res.* **17**, 439–460 (1967).
19. J. V. Alderson, J. B. Caress and R. L. Sager, The cooling rate of a glass fiber in the continuous filament process, Laboratory Report No. LR235, Pilkington Bros. Ltd., Lathom, Lancashire, U.K. (1968).
20. D. E. Bourne and D. G. Elliston, Heat transfer through the axially symmetric boundary layer on a moving circular fiber, *Int. J. Heat Mass Transfer* **13**, 583–590 (1970).
21. D. E. Bourne and H. Dixon, The cooling of fibers in the formation process, *Int. J. Heat Mass Transfer* **14**, 1323–1332 (1971).
22. S. L. Lee, T. S. Chen and B. F. Armaly, Mixed convection along isothermal vertical cylinders and needles, *Proc. Eighth Int. Heat Transfer Conf.*, Vol. 3, pp. 1425–1432 (1986).
23. S. L. Lee, T. S. Chen and B. F. Armaly, Non-parallel wave instability of mixed convection flow on inclined flat plates, *Int. J. Heat Mass Transfer* **31**, 1385–1398 (1988).
24. S. L. Lee, A new numerical formulation for parabolic differential equations under the consideration of large time steps, *Int. J. Numer. Meth. Engng* **26**, 1541–1549 (1988).

REFROIDISSEMENT D'UNE FEUILLE D'EPAISSEUR CONSTANTE ET MOBILE EN PRESENCE DE CONVECTION NATURELLE

Résumé—On étudie le refroidissement d'une feuille en mouvement continu en prenant en compte les forces de flottement. La distribution de température le long de l'interface solide–fluide est déterminée en résolvant un problème de transfert thermique conjugué de conduction dans la feuille et de convection mixte adjacente à la surface de la feuille. Pour une meilleure stabilité numérique, on emploie le schéma de fonction de pondération avec une transformation de coordonée axiale pour résoundre les équations transformées de couche limite. Il existe trois paramètres qui sont le nombre de Prandtl du fluide *Pr*, le paramètre de flottement Ω et le rapport des capacités thermiques *C*. Des résultats numériques incluant le nombre de Biot, la température de surface et le flux global transféré sont présentés pour $0{,}7 \leqslant Pr \leqslant 100$, $0 \leqslant \Omega \leqslant 10$ et $0{,}1 \leqslant C \leqslant 1$. La force de flottement a un effet sensible sur les résultats. Néanmoins le rapport des capacités thermiques est le paramètre le plus important. On conclut qu'avec un liquide, utilisé comme fluide de refroidissement, on obtient un meilleur refroidissement qu'avec un gaz, ceci parce que le liquide a une capacité thermique plus grande que le gaz. Le nombre de Prandtl n'a qu'un effet minime.

KÜHLUNG EINER STETIG BEWEGTEN PLATTE BEGRENZTER DICKE BEI NATÜRLICHER KONVEKTION

Zusammenfassung—In der vorliegenden Arbeit wird die Kühlung einer stetig bewegten Platte begrenzter Dicke untersucht. Dabei werden auch Auftriebseffekte berücksichtigt. Die Temperaturverteilung entlang einer fest-flüssigen Grenzfläche ergibt sich aus einem gekoppelten Wärmeübertragungsproblem. Innerhalb der Platte findet Wärmeleitung statt, an der Plattenoberfläche tritt erzwungene Mischkonvektion auf. Um die numerische Stabilität zu erhöhen, wird die Gewichtungsfunktion zusammen mit einer axialen Koordinatentransformation eingesetzt. Damit werden die transformierten Grenzschichtgleichungen gelöst. Die vorliegende Arbeit zeigt drei Einflußparameter auf: die Prandtl-Zahl des Fluids Pr, den Auftriebsparameter Ω und das Wärmekapazitätsverhältnis C. Die Biot-Zahl, die Oberflächentemperatur und der insgesamt von der Platte abgeführte Wärmestrom werden numerisch berechnet, und zwar für $0{,}7 \leqslant Pr \leqslant 100$, $0 \leqslant \Omega \leqslant 10$ und $0{,}1 \leqslant C \leqslant 1$. Es stellt sich heraus, daß die Auftriebskraft die Ergebnisse wesentlich beeinflußt. Der wichtigste Parameter ist jedoch das Wärmekapazitätsverhältnis. Die vorliegenden Ergebnisse lassen darauf schließen, daß Flüssigkeiten besser als Kühlmittel geeignet sind als Gase. Die Ursache hierfür liegt in der größeren Wärmekapazität von Flüssigkeiten. Die Prandtl-Zahl ist von untergeordneter Bedeutung.

ОХЛАЖДЕНИЕ ПРИ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ БЕСКОНЕЧНОЙ ДВИЖУЩЕЙСЯ ПЛАСТИНЫ КОНЕЧНОЙ ТОЛЩИНЫ

Аннотация—Исследуется охлаждение бесконечной движущейся пластины конечной толщины с учетом влияния подъемной силы. Распределение температур на границе раздела твердого тела и жидкости определяется путем решения сопряженной задачи теплопереноса, обусловленного теплопроводностью внутри пластины и смешанной свободной и вынужденной конвекцией около ее поверхности. Для улучшения устойчивости численного счета при решении преобразованных уравнений пограничного слоя используется схема весовых функций, а также трансформация осевой координаты. В данном исследовании установлены три существенных параметра, а именно, числа Прандтля жидкости Pr, параметра подъемной силы Ω и отношения теплоемкостей C. Численные результаты, определяющие числа Био, температуру поверхности и общую интенсивность теплопереноса пластины, представлены для $0{,}7 \leqslant Pr \leqslant 100$, $0 \leqslant \Omega \leqslant 10$ и $0{,}1 \leqslant C \leqslant 1$. Очевидно, что учет подъемной силы оказывает существенное влияние на результаты. Однако, наиболее важным параметром является отношение теплоемкостей. На основе данных результатов сделано заключение о том, что использование жидкости в качестве охлаждающей среды является более эффективным, чем использование газа, т.к. ее теплоемкость больше. Эффект числа Прандтля незначителен.